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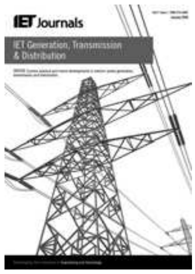

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Robust transmission system expansion considering planning uncertainties

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Abstract: This study presents a novel approach for transmission expansion planning (TEP) addressing the inherent uncertainties associated with the estimated investment costs of candidate transmission lines and the forecasted electricity demands during long-term planning horizon. The proposed TEP approach employs a renovated mixed integer linear programming formulation holding the optimality and low computation burden of linear modelling techniques. The above mentioned uncertainties are encountered through robust optimisation methodology enabling the system's planner to assess different levels of uncertainty and conservation throughout planning horizon. The proposed robust TEP procedure is successfully applied to Garver 6-bus, IEEE 30-bus and IEEE 118-bus test systems. Simulation results demonstrate that the uncertainty level of investment costs and electricity demands escalates the total expansion costs based on the scale of power system.

Nomenclature

Functions

$\Phi(\cdot)$	cumulative probability of standard normal distribution

Indices

i, j	index of different buses
l	index of different newly added circuits
m	index of different constraints
n	index of different decision variables
t_m	uncertain element of either objective function ($m = 0$) or mth constraint ($1 \leq m \leq DN$) adopting values from truncated uncertainty interval

Matrices and vectors

$\boldsymbol{A}$	DN × CN matrix of coefficients
$\boldsymbol{A}_{\text{aug}}$	DN × (CN + 1) augmented matrix of coefficients
$\boldsymbol{B}^{\text{prex}}$	suseptance matrix of pre-expansion transmission system
$\boldsymbol{B}^{\text{poex}}$	suseptance matrix of post-expansion transmission system
$\Delta\boldsymbol{B}$	suseptance matrix of newly added circuits
$\boldsymbol{C}$	CN-dimensional vector of coefficients
$\boldsymbol{C}_{\text{aug}}$	(CN + 1)-dimensional augmented vector of coefficients
$\boldsymbol{D}$	DN-dimensional vector of coefficients
$\boldsymbol{LB}$	CN-dimensional lower bound vector
$\boldsymbol{LB}_{\text{aug}}$	(CN + 1)-dimensional augmented lower bound vector
$\boldsymbol{P}_{\text{D}}$	vector of active power demands
$\boldsymbol{P}_{\text{G}}$	vector of active power generations
$\boldsymbol{UB}$	CN-dimensional upper bound vector
$\boldsymbol{UB}_{\text{aug}}$	(CN + 1)-dimensional augmented upper bound vector
$\boldsymbol{X}$	CN-dimensional decision vector
$\boldsymbol{X}^*$	optimal CN-dimensional decision vector
$\boldsymbol{X}_{\text{aug}}$	(CN + 1)-dimensional augmented decision vector
$\boldsymbol{\theta}$	vector of voltage angles

Parameters

$\tilde{a}_{mn}$	uncertain element of mth row and nth column in $\boldsymbol{A}$
$\bar{a}_{mn}$	estimated nominal value of $\tilde{a}_{mn}$
$\hat{a}_{mn}$	estimated range of $\tilde{a}_{mn}$
$\tilde{c}_n$	uncertain element of nth row in $\boldsymbol{C}$
$\bar{c}_n$	estimated nominal value of $\tilde{c}_n$
$\hat{c}_n$	estimated range of $\tilde{c}_n$
d_m	element of mth row in $\boldsymbol{D}$
lb_n	element of nth row in $\boldsymbol{LB}$
ub_n	element of nth row in $\boldsymbol{UB}$
BN	total number of buses
B_{ij}^{prex}	element of ith row and jth column in matrix $\boldsymbol{B}^{\text{prex}}$(mho)
CN	total number of decision variables
DN	total number of constraints
$F_{ij,\max}^{\text{prex}}$	maximum transferable power between buses i and j before expansion of transmission system (MW)

$F_{ij,\max}^{\text{poex}}$	maximum transferable power between buses i and j after expansion of transmission system (MW)
IC_{ij}	deterministic investment cost of a single circuit from bus i to bus j (M$)
$I\tilde{C}_{ij}$	uncertain investment cost of a single circuit from bus i to bus j (M$)
$I\bar{C}_{ij}$	nominal value of uncertain investment cost of a single circuit from bus i to bus j (M$)
$I\hat{C}_{ij}$	range of uncertain investment cost of a single circuit from bus i to bus j (M$)
$IC_{ij}^{\min}$	lower bound of uncertain investment cost interval of a single circuit from bus i to bus j (M$)
$IC_{ij}^{\max}$	upper bound of uncertain investment cost interval of a single circuit from bus i to bus j (M$)
$L_{ij,\max}$	maximum number of circuits which can be added from bus i to bus j
M	a sufficiently large positive number (e.g. $M = 10^3$)
P_{D_i}	deterministic demand of bus i (MW)
$\tilde{P}_{D_i}$	uncertain demand of bus i (MW)
$\bar{P}_{D_i}$	nominal value of demand of bus i (MW)
$\hat{P}_{D_i}$	range of demand of bus i (MW)
$P_{D_i}^{\min}$	lower bound of uncertain demand of bus i (MW)
$P_{D_i}^{\max}$	upper bound of demand of bus i (MW)
$P_{G_i,\min}$	minimum generation capacity in bus i (MW)
$P_{G_i,\max}$	maximum generation capacity in bus i (MW)
ΔB_{ij}	suseptance of a newly added circuit between buses i and j (mho)
$\Delta F_{ij,\max}$	capacity of a newly added circuit between buses i and j (MW)
$\lvert J_m \rvert$	number of elements of J_m
$\lvert S_m \rvert$	number of elements of S_{m}
α	degree of electricity demand uncertainty
β	degree of investment cost uncertainty
μ_{D_i}	mean value of uncertain demand in bus i (MW)
σ_{D_i}	standard deviation of uncertain demand in bus i (MW)
Γ_m	conservation level value for either objective function ($m = 0$) or mth constraint ($1 \le m \le \text{DN}$)
$\lfloor \Gamma_m \rfloor$	integer part of Γ_m (less than or equal to Γ_m)

Sets

J_m	set of uncertain elements of either objective function ($m = 0$) or mth constraint ($1 \le m \le \text{DN}$)
S_m	set of uncertain elements of either objective function ($m = 0$) or mth constraint ($1 \le m \le \text{DN}$) adopting values from respective uncertainty interval

Variables

p, y, z	continuous auxiliary robust modelling variables
x_n	element of nth row in $\boldsymbol{X}$
x_n^*	element of nth row in $\boldsymbol{X}^*$
$x_{\text{CN}+1}$	auxiliary decision variable to cope with right-hand side uncertainty
B_{ij}^{poex}	element of ith row and jth column in matrix $\boldsymbol{B}^{\text{poex}}$ (i.e. $B_{ij}^{\text{poex}} = B_{ij}^{\text{prex}} - L_{ij}\Delta B_{ij}$) (mho)
F_{ij}^{poex}	power flow between buses i and j after expansion of transmission system (MW)
L_{ij}	number of added circuits from bus i to bus j
L_{ij}^*	optimal number of added circuits from bus i to bus j
P_{G_i}	generation of bus i (MW)
εr_i	virtual generation increment for bus i
θ_i	voltage angle of bus i (rad)
θ_{slack}	voltage angle of slack bus (i.e. $\theta_{\text{slack}} = 0$) (rad)
ξ_{ij}^l	binary auxiliary modeling variable
Ω_{lt}	linear term of DC power flow equation
Ω_{nlt}	non-linear term of DC power flow equation
Ψ_{ij}	continuous auxiliary variable to recast a non-linear term (i.e. $L_{ij}\Delta B_{ij}\theta_j$) into a linear term

1 Introduction

The main objective of transmission expansion planning (TEP) is to acquire the optimum investment plan supplying the forecasted electricity demand as well as satisfying technical, financial, political and environmental constraints [1]. The well-known TEP approaches can be categorised into static and dynamic models. The static approaches determine the optimal location and number of new transmission lines to supply the forecasted demand adequately during a planning period. The dynamic approaches also determine the optimal installation timing of new transmission lines throughout the study period. The computational burden of dynamic approaches is more than static approaches. Since the static TEP models create an appropriate compromise between planning accuracy and computational tractability, this paper presents a renovated static planning tool.

Despite most of pioneering investigations conducted on deterministic TEP approaches [2], neglecting planning uncertainties may result in an expansion schedule that violates different technical and financial constraints. Therefore it is important to consider the multi-fold planning uncertainties during TEP studies. Non-deterministic TEP approaches cope with the planning uncertainties by means of

- Probabilistic and statistical models [3–12].
- Possibility and fuzzy models [4, 13].
- Interval numbers and interval models [14, 15].

A probabilistic TEP approach is presented in [3] considering the inherent uncertainties associated with the input data through probability distribution functions (PDF). As a result of utilising a large number of Monte Carlo simulations (MCS) throughout the planning process, the computational burden of this TEP approach is so high. Also, a combination of fuzzy multi-criteria decision making (MCDM), probabilistic optimal power flow and scenario technique is applied in [4] to cope with different aspects of planning uncertainties. In [5], MCS approach is employed to sample different scenarios using PDF and time series of the uncertain variables of the planning problem (e.g. the prices of supply and demand blocks). After evaluating sensitivities on transmission investments, the TEP model of [5] proposes the optimal investment plan via a MCDM approach. A probabilistic reliability-constrained TEP methodology is introduced in [6] taking into account the availability of generating units and transmission lines by an analytical approach. Besides, a sequential MCS approach is utilised in [7] to cope with the planning uncertainties associated with the available power of wind energy conversion systems as well as availability of generating units and transmission lines.

Some of non-deterministic TEP approaches employ the well-known stochastic programming (SP) method to obtain the optimal investment plan during a planning horizon [8–11]. In [8], the planning uncertainties associated with the forecasted load, equivalent availability of generating units and transmission capacity factor of transmission lines are addressed through an elegant SP methodology with probabilistic constraints representing the inherent uncertainty of available generating units and transmission lines. This TEP scheme also incorporates the mean–variance Markowitz theory into the optimisation problem to evaluate the risk of investment planning. In [9], a non-deterministic TEP model is presented which minimises the summation of investment and expected load shedding costs. The planning model of [9] considers the uncertainties associated with contingencies, load levels and inflows to hydro plants. A contingency screening strategy is also employed in [9] to reduce the computational burden of dealing with a large number of contingencies in a practical power system. In [10], a vulnerability constrained TEP approach is proposed as a SP model with recourse minimising expected load shedding level and expected load shedding cost in conjunction with the investment cost. The inherent uncertainty associated with deliberate outages of terrorist attacks is accounted through different scenarios and vulnerability analysis. A stochastic coordinated generation and TEP model is introduced in [11] utilising a lattice MCS and a scenario reduction technique to simulate planning uncertainties including random outages of generating units and transmission lines along with inaccuracy in long-term load forecasting. In addition to the abovementioned TEP approaches applying SP models, a chance constrained TEP method is proposed in [12] to consider wind power forecast and load forecast uncertainties. The planning tool of [12] presents a combination of MCS and analytical probabilistic load flow analysis method to deal with the planning uncertainties.

In [13], a fuzzy integer programming model is presented to cope with the uncertainty of the investment budget and the delivery marginal rate of the system. The approach of [13] makes it possible to employ expert experiences throughout solving the TEP problem. In [14], the inherent demand uncertainty is modelled by two mixed integer non-linear programming formulations utilising an interval restricted between the maximum and minimum demand. This interval represents the abovementioned uncertainty in the system as a whole and in each load bus as a part of the system. Both of the planning models determine the total demand value which can be supplied throughout a specific optimal investment plan. In [15], the minimum load cutting problem associated with planning studies under uncertain demand is formulated as a bi-level linear programming (LP) problem. The bi-level LP model of [15] considers the uncertain demand as a bounded interval like that of [14] and presents the minimum load cutting value as a bounded interval. Therefore the planning safety under different expansion scenarios can be evaluated through the upper bound of the minimum load cutting interval.

Most of the abovementioned TEP methodologies are intrinsically large and complex. The non-deterministic planning approaches usually lead to numerically intractable models for practical power systems which are also incapable to control the conservation level (CL) of the optimal investment plan. Therefore it is appealing to introduce a straightforward TEP methodology coping with the planning uncertainties via a tractable mathematical model. A more recent methodology for optimisation under uncertainty is robust optimisation (RO) which models data uncertainties through a set of deterministic and bounded intervals [16]. The RO approach solves a deterministic version of the original uncertain problem (i.e. robust counterpart) to acquire an optimal solution which is immunised against data uncertainties [17]. Among a wide range of literature toward the RO framework, a numerically tractable MILP robust formulation is introduced in [18] for dealing with discrete optimisation under uncertainty. This methodology is used successfully in several areas of power systems recently [19–21]. A RO-based solution approach is introduced in [19] to acquire hourly bidding curves for a price-taker electricity producer trading in a pool market. The model of [19] employs price confidence intervals instead of price forecast data sets and solves a set of robust MILP problems providing optimal bidding curves. In [20], the contingency constrained single-bus unit commitment (CCUC) problem with n–k security criterion is modelled using the concept of the RO methodology. The most attractive feature of this CCUC approach in comparison with previous models is its computational tractability apart from the size of credible contingencies. In [21], a robust MILP framework is proposed to obtain an environmentally and economically optimal integration plan for plug-in hybrid electric vehicles into the power grid considering integration uncertainties. However, to the best of the authors' knowledge, RO has never been used in the previous TEP research works.

The main contributions of this paper can be summarised as follows:

- Introducing a renovated TEP approach as a MILP optimisation problem, resolving the problem of isolated buses, and modelling investment cost and load forecast uncertainties.
- Applying the RO methodology throughout TEP studies as a tractable optimisation technique to cope with the planning uncertainties.
- Considering different levels of the solution conservation using a priori chance constraints. The proposed TEP model deals with the innate planning uncertainties through distribution-free restricted intervals and thus it can be used even if the distribution of the uncertain variables is not available.

The rest of this paper is organised as follows. The proposed TEP approach and its robust counterpart, designated as robust TEP (RTEP) method, are introduced in Section 2. In Section 3, an illustrative example is provided for the well-known Garver test system to demonstrate the effectiveness of the suggested RTEP model. To evaluate the computational tractability of the proposed approach, the RTEP methodology is tested on IEEE 30-bus and IEEE 118-bus test systems in Section 4. Finally, the main conclusions are drawn in Section 5.

2 Transmission system expansion planning

2.1 Deterministic TEP model

In this section, a new MILP formulation for TEP problem will be introduced, which is capable to cope with the problem of disconnected buses straightforwardly. TEP problem is resolved previously in terms of a non-linear methodology to deal with the problem of isolated networks [22]. In this paper, the proposed TEP approach, as an intrinsically non-linear formulation, will be linearised by applying some slight modifications. The least-cost objective function of

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TEP problem can be expressed as follows

$$\text{Min} \quad \sum_{i=1}^{\mathrm{BN}-1} \sum_{j=i+1}^{\mathrm{BN}} IC_{ij}\, L_{ij} \tag{1}$$

The objective function (1) states minimisation of total expansion costs during a predefined planning horizon. As depicted in (1), the circuits of each corridor are assumed to be identical for the sake of simplicity. To ensure the nodal power balance for each bus of the post-expansion transmission system, the well-known DC power flow equation is incorporated into the proposed TEP problem explicitly. The DC power flow equation imposes the following relation for the post-expansion transmission system

$$\boldsymbol{B}^{\mathrm{poex}}\, \boldsymbol{\theta} = \left(\boldsymbol{B}^{\mathrm{prex}} + \Delta \boldsymbol{B}\right)\, \boldsymbol{\theta} = \boldsymbol{P}_{\mathrm{G}} - \boldsymbol{P}_{\mathrm{D}} \tag{2}$$

Consequently, the nodal power balance equation for bus i can be obtained through multiplying ith row of matrix $\boldsymbol{B}^{\mathrm{poex}}$ and vector $\boldsymbol{\theta}$ in (2) as follows (see (3))

About (3), note that according to the rules of constructing admittance matrix, $-L_{ij}\Delta B_{ij}$ represents the augmented term of mutual admittance of bus i because of newly added circuits from bus i to bus j, and negative summation of all the augmented mutual admittances of bus i represents the augmented term of self-admittance of bus i. As can be observed in (3), there is a linear term Ω_{lt} related to the pre-expansion transmission system and also a non-linear term Ω_{nlt} related to the expanded transmission system. Owing to multiplication of unknown integer (i.e. $L_{ij} \in Z$) and continuous variables (i.e. $\theta_i \in R$ or $\theta_j \in R$), Ω_{nlt} becomes a non-linear term. In this paper, big M linearisation technique [23] is employed to resolve the non-linearity of (3) in terms of continuous and binary auxiliary variables providing a set of new constraints to imitate the characteristic of non-linear terms. Therefore a new continuous auxiliary variable is defined as follows

$$\Psi_{ij} = L_{ij}\, \Delta B_{ij}\, \theta_j \tag{4}$$

Furthermore, a binary auxiliary variable ξ_{ij}^{l} is introduced in (5)–(10) to ensure the characteristic of (4) for different number of newly added circuits from bus i to bus j

$$\Psi_{ij} + M\, \xi_{ij}^{l} \leq l\, \Delta B_{ij}\, \theta_j + M, \quad l = 1, \ldots, L_{ij,\max};\ i = 1, \ldots, \mathrm{BN}-1, \quad j = i+1, \ldots, \mathrm{BN} \tag{5}$$

$$\Psi_{ij} - M\, \xi_{ij}^{l} \geq l\, \Delta B_{ij}\, \theta_j - M, \quad l = 1, \ldots, L_{ij,\max};\ i = 1, \ldots, \mathrm{BN}-1, \quad j = i+1, \ldots, \mathrm{BN} \tag{6}$$

$$\Psi_{ij} \leq M \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^{l}, \quad i = 1, \ldots, \mathrm{BN}-1,\ j = i+1, \ldots, \mathrm{BN} \tag{7}$$

$$\Psi_{ij} \geq -M \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^{l}, \quad i = 1, \ldots, \mathrm{BN}-1,\ j = i+1, \ldots, \mathrm{BN} \tag{8}$$

$$L_{ij} = \sum_{l=1}^{L_{ij,\max}} l\, \xi_{ij}^{l}, \quad i = 1, \ldots, \mathrm{BN}-1,\ j = i+1, \ldots, \mathrm{BN} \tag{9}$$

$$\sum_{l=1}^{L_{ij,\max}} \xi_{ij}^{l} \leq 1, \quad i = 1, \ldots, \mathrm{BN}-1,\ j = i+1, \ldots, \mathrm{BN} \tag{10}$$

According to big M linearisation technique, the non-linear terms (e.g. $L_{ij}\ \Delta B_{ij}\ \theta_j$) are replaced by the continuous auxiliary variables, that is, Ψ_{ij} shown in (4), and the binary auxiliary variables, that is, ξ_{ij}^{l} shown in (5)–(10), model different states of the non-linear terms because of different values adopted by the integer variables L_{ij}. Three components contribute to the auxiliary variable Ψ_{ij} shown in (4). The first one is the number of newly added circuits from bus i to bus j designated as L_{ij}, which is an integer decision variable. The second component of Ψ_{ij} is ΔB_{ij} representing the susceptance of each newly added circuit from bus i to bus j. Therefore ΔB_{ij} is a constant term in Ψ_{ij} because the susceptance is known based on the characteristics of candidate corridors. Finally, θ_j is the third component of Ψ_{ij} representing a continuous variable. Therefore the unknown components of Ψ_{ij} are L_{ij} and θ_j, which can be obtained by solving the optimisation problem of deterministic TEP formulation.

Based on (10), either all binary variables ξ_{ij}^{l} ($l = 1, \ldots, L_{ij,\max}$) are zero or one of them is one while the other ξ_{ij}^{l} variables are zero. If $L_{ij} = 0$ (i.e. no new circuit between buses i and j is added), all ξ_{ij}^{l} variables become zero according to (9) and also (7) and (8) enforce $\Psi_{ij} = 0$. In this

$$\begin{aligned}
\left[B_{i1}^{\mathrm{poex}} \quad \ldots \quad B_{ii}^{\mathrm{poex}} \quad \ldots \quad B_{in}^{\mathrm{poex}}\right] \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_i \\ \vdots \\ \theta_n \end{bmatrix} &= \left(B_{i1}^{\mathrm{prex}} - L_{i1}\, \Delta B_{i1}\right)\, \theta_1 + \cdots - \sum_{\substack{j=1 \\ j \neq i}}^{\mathrm{BN}} \left[\left(B_{ij}^{\mathrm{prex}} - L_{ij}\, \Delta B_{ij}\right)\, \theta_i\right] + \cdots + \left(B_{in}^{\mathrm{prex}} - L_{in}\, \Delta B_{in}\right)\theta_n \\
&= \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{\mathrm{BN}} \left[B_{ij}^{\mathrm{prex}}\left(\theta_j - \theta_i\right)\right]}_{\Omega_{\mathrm{lt}}} + \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{\mathrm{BN}} \left[L_{ij}\, \Delta B_{ij}\, \left(\theta_i - \theta_j\right)\right]}_{\Omega_{\mathrm{nlt}}} = P_{G_i} - P_{D_i}, \quad i = 1, \ldots, \mathrm{BN}
\end{aligned} \tag{3}$$

case, constraints (5) and (6) for all values of l ($l = 1, \ldots, L_{ij,\max}$) are neutralised. However, if $0 < L_{ij} \le L_{ij,\max}$, then $\xi_{ij}^{l=L_{ij}} = 1$ and the other ξ_{ij}^{l} variables become zero based on (9) and (10). In this case, the constraints (7) and (8) as well as (5) and (6) for $l \neq L_{ij}$ are neutralised, whereas (5) and (6) for $l = L_{ij}$ results in $\Psi_{ij} = l\Delta B_{ij}\theta_j$ ensuring (4). Note that multiplication of l and θ_j in (9) and (10) does not cause non-linearity of the formulation, since l is an integer index and not a variable, despite L_{ij}. A concise numerical example is given in Appendix 1 to clarify the task of big M linearisation technique to resolve the non-linearity of $L_{ij}\,\Delta B_{ij}\,\theta_j$ in terms of Ψ_{ij} and ξ_{ij}^{l} ($l = 1, \ldots, L_{ij,\max}$). Thus, the non-linear term Ω_{nlt} in (3) can be replaced with the following linear term

$$\sum_{\substack{j=1\\ j\neq i}}^{\text{BN}} \left[L_{ij}\,\Delta B_{ij}\left(\theta_i - \theta_j\right)\right] = \sum_{\substack{j=1\\ j\neq i}}^{\text{BN}} \left(\Psi_{ji} - \Psi_{ij}\right), \quad i = 1, \ldots, \text{BN} \tag{11}$$

Considering (2), (3) and (11), the nodal power balance equation for bus i can be expressed as follows (see (12))

The power flow equation from bus i to bus j and its operational limitations can be written as below (see (13))

$$\left|F_{ij}^{\text{poex}}\right| \le F_{ij,\max}^{\text{poex}} = F_{ij,\max}^{\text{prex}} + L_{ij}\,\Delta F_{ij,\max}, \quad i = 1, \ldots, \text{BN}-1,\ j = i+1, \ldots, \text{BN} \tag{14}$$

The generation capacity limitations for bus i can be given by (15)

$$P_{G_i,\min} \le P_{G_i} \le P_{G_i,\max}, \quad i = 1, \ldots, \text{BN} \tag{15}$$

Consequently, the whole optimisation problem in the deterministic case can be written as follows (see (16))

The main appealing feature of this TEP model is its linear formulation, which can be solved easily and efficiently through commercial software packages for optimisation. Next section will present a robust counterpart for this deterministic TEP formulation.

$$\begin{bmatrix} B_{i1}^{\text{poex}} & \ldots & B_{ii}^{\text{poex}} & \ldots & B_{in}^{\text{poex}} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_i \\ \vdots \\ \theta_n \end{bmatrix} = \sum_{\substack{j=1\\ j\neq i}}^{BN} \left[B_{ij}^{\text{prex}}\left(\theta_j - \theta_i\right) + \left(\Psi_{ji} - \Psi_{ij}\right)\right] = P_{G_i} - P_{D_i}, \quad i = 1, \ldots, \text{BN} \tag{12}$$

$$\begin{aligned} F_{ij}^{\text{poex}} = -B_{ij}^{\text{poex}}\left(\theta_i - \theta_j\right) &= \left(-B_{ij}^{\text{prex}} + L_{ij}\,\Delta B_{ij}\right)\left(\theta_i - \theta_j\right) \\ &= -B_{ij}^{\text{prex}}\left(\theta_i - \theta_j\right) + \Psi_{ji} - \Psi_{ij}, \\ &\quad i = 1, \ldots, \text{BN}-1,\ j = i+1, \ldots, \text{BN} \end{aligned} \tag{13}$$

$$\begin{aligned} &\text{Min} \quad \sum_{i=1}^{\text{BN}-1} \sum_{j=i+1}^{\text{BN}} IC_{ij}\,L_{ij} \\ &\text{s.t.} \quad \sum_{\substack{j=1\\ j\neq i}}^{\text{BN}} \left[B_{ij}^{\text{prex}}\left(\theta_j - \theta_i\right) + \left(\Psi_{ji} - \Psi_{ij}\right)\right] = P_{G_i} - P_{D_i}, \quad i = 1, \ldots, \text{BN} \\ &\qquad \left|-B_{ij}^{\text{prex}}\left(\theta_i - \theta_j\right) + \Psi_{ji} - \Psi_{ij}\right| \le F_{ij,\max}^{\text{poex}}, \quad i = 1, \ldots, \text{BN}-1, \quad j = i+1, \ldots, \text{BN} \\ &\qquad \Psi_{ij} + M\,\xi_{ij}^{l} \le l\,\Delta B_{ij}\,\theta_j + M, \quad l = 1, \ldots, L_{ij,\max};\, i = 1, \ldots, \text{BN}-1, \quad j = i+1, \ldots, \text{BN} \\ &\qquad \Psi_{ij} - M\,\xi_{ij}^{l} \ge l\,\Delta B_{ij}\,\theta_j - M, \quad l = 1, \ldots, L_{ij,\max};\, i = 1, \ldots, \text{BN}-1, \quad j = i+1, \ldots, \text{BN} \\ &\qquad \left|\Psi_{ij}\right| \le M \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^{l}, \quad i = 1, \ldots, \text{BN}-1, \quad j = i+1, \ldots, \text{BN} \\ &\qquad L_{ij} = \sum_{l=1}^{L_{ij,\max}} l\,\xi_{ij}^{l}, \quad \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^{l} \le 1, \quad i = 1, \ldots, \text{BN}-1, \quad j = i+1, \ldots, \text{BN} \\ &\qquad P_{G_i,\min} \le P_{G_i} \le P_{G_i,\max}, \quad i = 1, \ldots, \text{BN} \\ &\qquad \theta_{\text{slack}} = 0, \qquad P_{G_i}, \theta_i \in R, \quad i = 1, \ldots, \text{BN}, \qquad L_{ij} \in Z,\ \Psi_{ij} \in R, \quad i = 1, \ldots, \text{BN}-1,\ j = i+1, \ldots, \text{BN} \\ &\qquad \xi_{ij}^{l} \in \{0, 1\}, \quad l = 1, \ldots, L_{ij,\max};\, i = 1, \ldots, \text{BN},\ j = i+1, \ldots, \text{BN} \end{aligned} \tag{16}$$

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2.2 Robust TEP model

The deterministic TEP model described in the preceding section can be rewritten compactly for notation brevity as follows

$$\begin{aligned} &\text{Min} \quad \boldsymbol{C}^{\mathrm{T}} \boldsymbol{X} \\ &\text{s.t.} \quad \boldsymbol{AX} \le \boldsymbol{D}, \quad \boldsymbol{LB} \le \boldsymbol{X} \le \boldsymbol{UB} \\ &x_n \in Z, n = 1, 2, \ldots, k; \quad x_n \in R, n = k+1, k+2, \ldots, \mathrm{CN} \end{aligned} \tag{17}$$

The major concern of the RTEP methodology in this paper is to present an optimal investment schedule that is robust with regard to data uncertainties associated with the estimated investment costs of candidate transmission lines and the forecasted electricity demand throughout long-term planning horizon (i.e. vectors $\boldsymbol{C}$ and $\boldsymbol{D}$ in (17), respectively). Without loss of generality, suppose that only the elements of vector $\boldsymbol{C}$, and matrix $\boldsymbol{A}$ are subject to data uncertainty. Likewise, a new auxiliary variable $x_{\mathrm{CN}+1}$ can be introduced to cope with data uncertainty associated with vector $\boldsymbol{D}$ through recasting (17) as follows (see (18))

where

$$\begin{aligned} &\boldsymbol{C}_{\text{aug}} = \begin{bmatrix} \boldsymbol{C} \\ 0 \end{bmatrix}, \quad \boldsymbol{X}_{\text{aug}} = \begin{bmatrix} \boldsymbol{X} \\ x_{\mathrm{CN}+1} \end{bmatrix}, \\ &\boldsymbol{A}_{\text{aug}} = [\boldsymbol{A} \quad -\boldsymbol{D}], \quad \boldsymbol{LB}_{\text{aug}} = \begin{bmatrix} \boldsymbol{LB} \\ 1 \end{bmatrix}, \\ &\boldsymbol{UB}_{\text{aug}} = \begin{bmatrix} \boldsymbol{UB} \\ 1 \end{bmatrix} \end{aligned} \tag{19}$$

As mentioned earlier, the RO methodology models data uncertainties through bounded intervals designated as uncertainty sets [16]. Therefore the uncertain elements of vector $\boldsymbol{C}$ and matrix $\boldsymbol{A}$ in (17) can be represented using nominal value and range of each uncertain element as follows

$$\tilde{c}_n = [\bar{c}_n - \hat{c}_n, \bar{c}_n + \hat{c}_n] \quad \tilde{c}_n \in \boldsymbol{C} \tag{20}$$

$$\tilde{a}_{mn} = [\bar{a}_{mn} - \hat{a}_{mn}, \bar{a}_{mn} + \hat{a}_{mn}] \quad \tilde{a}_{mn} \in \boldsymbol{A} \tag{21}$$

Since the RO framework presents an optimal solution which is immunised against all data uncertainties by solving the worst-case problem, the resulting solution almost gives up optimality to guarantee robustness. In this paper, a well-designed robust formulation is exploited to conquer the problem of over-conservatism associated with early versions of the RO methodology [18]. Consequently, CL, denoted by Γ_m ($m = 0, 1, \ldots, \mathrm{DN}$), is introduced in [18] as a varying number adopting different values in the interval $[0,|J_m|]$ to control the CL of the robust solution pertaining to data uncertainties. J_m represents a set including the uncertain elements of either the objective function ($m = 0$), $J_0 = \{n|\hat{c}_n > 0\}$ or mth constraint ($m = 1, \ldots, \mathrm{DN}$), $J_m = \{n|\hat{a}_{mn} > 0\}$. Therefore Γ_0 and Γ_m represent the CL value for the objective function and mth constraint, respectively. As it is implausible that all of the non-deterministic elements of either the objective function or mth constraint change simultaneously, the proposed methodology assumes that up to $\lfloor \Gamma_m \rfloor$ of these elements are allowed to adopt values among their respective ranges as depicted in (20) and (21), and one of these uncertain elements is allowed to adopt values among a truncated range as follows

$$\begin{aligned} &\tilde{c}_{t_0} = \left[\bar{c}_{t_0} - (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0}, \bar{c}_{t_0} + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0}\right], \\ &\quad \tilde{c}_{t_0} \in J_0, \ m = 0 \end{aligned} \tag{22}$$

$$\begin{aligned} &\tilde{a}_{mt_m} = \left[\bar{a}_{mt_m} - (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m}, \bar{a}_{mt_m} + (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m}\right], \\ &\quad \tilde{a}_{mt_m} \in J_m, \ m = 1, 2, \ldots, \mathrm{DN} \end{aligned} \tag{23}$$

In this way, CL can adopt real values in addition to integer ones. For instance, $\Gamma_m = 2.5$ means that two non-deterministic elements of the objective function ($m = 0$) or mth constraint ($m = 1, \ldots, \mathrm{DN}$) can vary in the full ranges and one uncertain element can vary in the half range. The robust counterpart of (17) can be represented by (24) (see (24))

This non-linear robust counterpart can be rewritten as a MILP problem by applying duality theory and linearisation

$$\begin{aligned} &\text{Min} \quad \boldsymbol{C}_{\text{aug}}^{\mathrm{T}} \boldsymbol{X}_{\text{aug}} \\ &\text{s.t.} \quad \boldsymbol{A}_{\text{aug}} \boldsymbol{X}_{\text{aug}} \le 0, \quad \boldsymbol{LB}_{\text{aug}} \le \boldsymbol{X}_{\text{aug}} \le \boldsymbol{UB}_{\text{aug}} \\ &x_n \in Z, n = 1, 2, \ldots, k; \quad x_n \in R, n = k+1, k+2, \ldots, \mathrm{CN}, \mathrm{CN}+1 \end{aligned} \tag{18}$$

$$\begin{aligned} &\text{Min} \quad \sum_n \bar{c}_n x_n + \underset{\{S_0 \cup \{t_0\} | S_0 \subseteq J_0, |S_0| = \lfloor \Gamma_0 \rfloor, t_0 \in J_0 \setminus S_0\}}{\text{Max}} \left\{ \sum_{n \in S_0} \hat{c}_n |x_n| + (\Gamma_0 - \lfloor \Gamma_0 \rfloor)\hat{c}_{t_0} \left|x_{t_0}\right| \right\} \\ &\text{s.t.} \quad \sum_n \bar{a}_{mn} x_n + \underset{\{S_m \cup \{t_m\} | S_m \subseteq J_m, |S_m| = \lfloor \Gamma_m \rfloor, t_m \in J_m \setminus S_m\}}{\text{Max}} \left\{ \sum_{n \in S_m} \hat{a}_{mn} |x_n| + (\Gamma_m - \lfloor \Gamma_m \rfloor)\hat{a}_{mt_m} \left|x_{t_m}\right| \right\} \le d_m \quad m = 1, 2, \ldots, \mathrm{DN} \\ &\quad lb_n \le x_n \le ub_n, \quad n = 1, 2, \ldots, \mathrm{CN} \\ &\quad x_n \in Z, \ n = 1, 2, \ldots, k; \quad x_n \in R, \ n = k+1, k+2, \ldots, \mathrm{CN} \end{aligned} \tag{24}$$

techniques for non-linear terms of (24) as given by (25)

$$
\begin{aligned}
&\text{Min} \quad \sum_n \bar{c}_n x_n + z_0 \Gamma_0 + \sum_{n\in J_0} p_{0n} \\
&\text{s.t.} \quad \sum_n \bar{a}_{mn} x_n + z_m \Gamma_m + \sum_{n\in J_m} p_{mn} \le d_m, \quad m=1,2,\ldots,\text{DN} \\
&\quad z_0 + p_{0n} \ge \hat{c}_n y_n \quad \forall n \in J_0 \\
&\quad z_m + p_{mn} \ge \hat{a}_{mn} y_n \quad \forall n \in J_m, \quad m=1,2,\ldots,\text{DN} \\
&\quad -y_n \le x_n \le y_n, \quad n=1,2,\ldots,\text{CN} \\
&\quad lb_n \le x_n \le ub_n, \quad n=1,2,\ldots,\text{CN} \\
&\quad p_{mn} \ge 0 \quad \forall n \in J_m, \quad m=0,1,2,\ldots,\text{DN} \\
&\quad y_n \ge 0 \quad n=1,2,\ldots,\text{CN}, \quad z_m \ge 0, \quad m=0,1,2,\ldots,\text{DN} \\
&\quad x_n \in Z,\ n=1,2,\ldots,k; \quad x_n \in R, \quad n=k+1,k+2,\ldots,\text{CN}
\end{aligned} \tag{25}
$$

The optimal solution of (25) designated as $\boldsymbol{X}^*$ is only feasible for a set of predefined CL values. Provided that the uncertain elements of (17) vary between their respective bounded intervals as demonstrated by (21), the chance of constraint violation, regarding $\boldsymbol{X}^*$, can be calculated as follows [18]

$$
P\left[\sum_n \tilde{a}_{mn} x_n^* > d_m\right] \le \exp\left[-\frac{\Gamma_m^2}{2|J_m|}\right], \quad m=1,2,\ldots,\text{DN} \tag{26}
$$

Therefore the chance of constraint violation is a priori chance constraint which can be calculated easily before finding the optimal solution of (25) using Γ_m and J_m (i.e. $m=1, \ldots, \text{DN}$). Now, the RTEP model can be extracted through introducing a set of symmetric bounded intervals which represent the inherent uncertainty associated with the estimated investment costs of candidate transmission lines as well as the forecasted electricity demand as follows

$$
\tilde{IC}_{ij} = \left[\bar{IC}_{ij} - \hat{IC}_{ij}, \bar{IC}_{ij} + \hat{IC}_{ij}\right], \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \tag{27}
$$

$$
\tilde{P}_{D_i} = \left[\bar{P}_{D_i} - \hat{P}_{D_i}, \bar{P}_{D_i} + \hat{P}_{D_i}\right], \quad i=1,\ldots,\text{BN} \tag{28}
$$

According to the robust counterpart of (17), represented by (25), the RTEP formulation for (16) is given by (29) (see (29))

The nodal power balance constraint for bus i (i.e. $i=1, \ldots, \text{BN}$) only incorporates one uncertain parameter which is electricity demand. Therefore $|J_i|$ is equal to one and Γ_i can adopt different real values between [0, 1]. When Γ_i adopts the largest possible value (i.e. $\Gamma_i = 1$), the optimal investment plan is immunised against electricity demand uncertainty entirely. Although (26) denotes that the chance of constraint violation for the worst-case RTEP problem (i.e. $|J_i| = \Gamma_i = 1$) is equal to or less than 0.60653, the accurate chance is equal to zero given that the uncertain nodal power demand for each bus changes within its respective range depicted by (28). As a result, engendering an accurate chance of

$$
\begin{aligned}
&\text{Min} \quad \sum_{i=1}^{\text{BN}-1} \sum_{j=i+1}^{\text{BN}} \bar{IC}_{ij} L_{ij} + z_0 \Gamma_0 + \sum_{i=1}^{\text{BN}-1} \sum_{j=i+1}^{\text{BN}} p_{0ij} \\
&\text{s.t.} \quad \sum_{\substack{j=1\\ j\ne i}}^{\text{BN}} \left[B_{ij}^{\text{prex}}\left(\theta_j - \theta_i\right) + \left(\Psi_{ji} - \Psi_{ij}\right)\right] + z_i \Gamma_i + p_i = P_{G_i} - \bar{P}_{D_i}, \quad i=1,\ldots,\text{BN} \\
&\quad \left|-B_{ij}^{\text{prex}}\left(\theta_i - \theta_j\right) + \Psi_{ji} - \Psi_{ij}\right| \le F_{ij,\max}^{\text{poex}}, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad \Psi_{ij} + M\,\xi_{ij}^l \le l\,\Delta B_{ij}\,\theta_j + M, \quad l=1,\ldots,L_{ij,\max};\ i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad \Psi_{ij} - M\,\xi_{ij}^l \ge l\,\Delta B_{ij}\,\theta_j - M, \quad l=1,\ldots,L_{ij,\max};\ i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad \left|\Psi_{ij}\right| \le M \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^l, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad L_{ij} = \sum_{l=1}^{L_{ij,\max}} l\,\xi_{ij}^l, \quad \sum_{l=1}^{L_{ij,\max}} \xi_{ij}^l \le 1, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad z_0 + p_{0ij} \ge \hat{IC}_{ij}\, y_{IC_{ij}}, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad z_i + p_i \ge \hat{P}_{D_i}\, y_{P_{D_i}}, \quad i=1,\ldots,\text{BN} \\
&\quad L_{ij} \le y_{IC_{ij}}, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN}, \quad 1 \le y_{P_{Di}}, \quad i=1,\ldots,\text{BN} \\
&\quad p_{0ij} \ge 0, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN}, \quad p_i \ge 0, \quad i=1,\ldots,\text{BN}, \quad z_0 \ge 0, \quad z_i \ge 0 \quad i=1,\ldots,\text{BN} \\
&\quad P_{G_i,\min} \le P_{G_i} \le P_{G_i,\max}, \quad i=1,\ldots,\text{BN} \\
&\quad \theta_{\text{slack}} = 0, \quad P_{G_i}, \theta_i \in R, \quad i=1,\ldots,\text{BN}, \quad L_{ij} \in Z, \Psi_{ij} \in R, \quad i=1,\ldots,\text{BN}-1, \quad j=i+1,\ldots,\text{BN} \\
&\quad \xi_{ij}^l \in \{0,1\}, \quad l=1,\ldots,L_{ij,\max};\ i=1,\ldots,\text{BN}, \quad j=i+1,\ldots,\text{BN}
\end{aligned} \tag{29}
$$

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constraint violation is pleasing for the proposed RTEP methodology. Moreover, the chance of constraint violation for the proposed RTEP model when the uncertain nodal power demand varies within its entire domain can be addressed in terms of distributional chance constrains incorporating the probability distribution of the uncertain demand. Regardless of the probability distribution of estimated investment costs which are ineffective in the chance of constraint violation of the proposed RTEP model, the inherent uncertainty of electricity demand, as the only uncertain parameter of the nodal power balance constraint for each load bus, can be modeled through normal probability distribution effectively [24]. As depicted in Fig. 1, the probability distribution of uncertain demand for each load bus can be identified via its mean value μ_{D_i} and standard deviation σ_{D_i}. Given that the nominal value of uncertain electricity demand interval corresponds to the mean value of its normal probability distribution, the chance of power balance violation for each bus can be controlled through the range of uncertain electricity demand interval enveloping the normal probability distribution of demand symmetrically as shown in Fig. 1. The chance of power balance violation for each bus, regarding the range of uncertain electricity demand interval, can be calculated as follows

$$P\left[\sum_{\substack{j=1\\ j\neq i}}^{\mathrm{BN}}\left[B_{ij}^{\mathrm{prex}}\left(\theta_j-\theta_i\right)+\left(\Psi_{ji}-\Psi_{ij}\right)\right]\neq P_{G_i}-\tilde{P}_{D_i}\right]$$
$$\leq 1-\Phi\left[\frac{\left(\bar{P}_{D_i}+\Gamma_i\hat{P}_{D_i}\right)-\mu_{D_i}}{\sigma_{D_i}}\right],$$
$$i=1,\ldots,\mathrm{BN} \tag{30}$$

As depicted in Fig. 1, the chance of nodal power balance violation is equal to the shaded area of normal probability distribution that depends on the values of Γ_i and $\hat{P}_{D_i}$ when the value of σ_{D_i} is known. For instance, given that $\Gamma_i=1$, $\bar{P}_{D_i}=\mu_{D_i}$ and $\hat{P}_{D_i}=3\sigma_{D_i}$, the chance of nodal power balance violation is equal to or less than 0.00135. The introduced formulation by (29) is a MILP problem that can be solved effectively by branch and bound algorithm even for large-scale power systems. Next section provides an illustrative example to elucidate the detailed performance of this method.

3 Illustrative example

In this section, the proposed RTEP model is implemented on the well-known Garver test system as depicted in Fig. 2 [25] where the total generating capacity and the total load demand of the original test system are tripled. The simulations have been performed with CPLEX solver in generalized algebraic modelling systems (GAMS) environment on a 64-bit computer with 6 GB of RAM and Intel Core i7 CPU. The characteristics of candidate corridors for this example are tabulated in Table 1. Although Garver test system includes one disconnected bus (i.e. bus 6 shown in Fig. 2), the proposed RTEP methodology in (29) is capable to resolve this issue easily. In this illustrative example, the main concern is given to the impact of uncertainty degree on the outputs resulted from the RTEP method. Here, the uncertain electricity demands and the uncertain investment costs have been

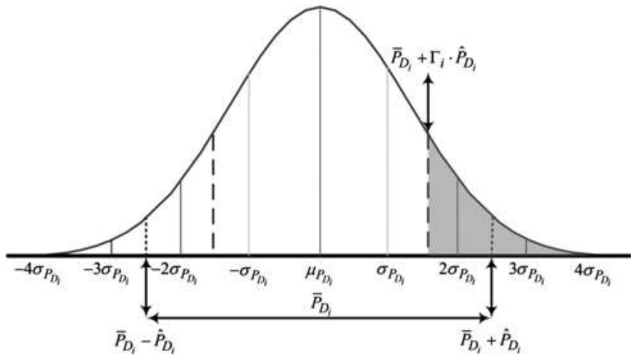

Fig. 1 *Normal probability distribution against bounded interval of uncertain electricity demand*

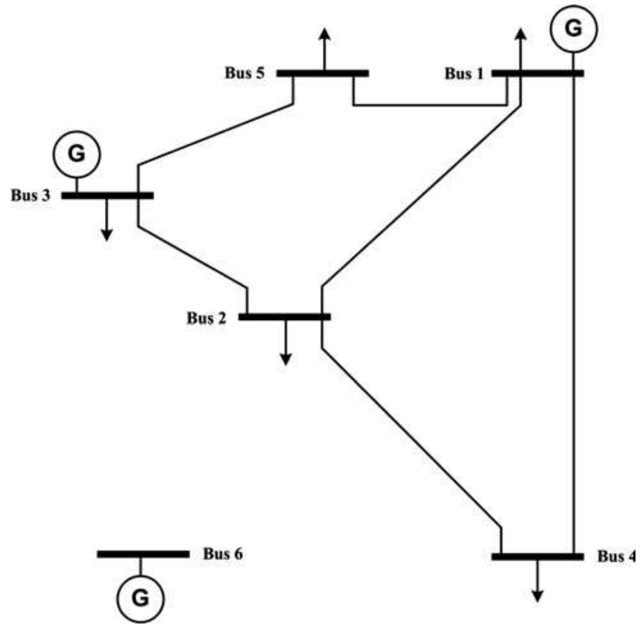

Fig. 2 *Garver test system*

Table 1 Characteristics of candidate corridors for Garver test system

From bus	To Bus	Capacity, MW	Reactance, pu	Cost, M$
1	2	80	j0.05	10
1	3	50	j0.04	8
1	4	100	j0.10	14
1	5	120	j0.067	18
1	6	200	j0.20	25
2	3	80	j0.05	10
2	4	120	j0.067	18
2	5	100	j0.10	14
2	6	200	j0.20	25
3	4	80	j0.05	10
3	5	80	j0.05	10
3	6	100	j0.10	14
4	5	100	j0.10	14
4	6	50	j0.04	8
5	6	50	j0.04	8

considered in terms of symmetric bounded intervals as follows

$$P_{D_i}^{\min}=(1-\alpha)\bar{P}_{D_i},\quad P_{D_i}^{\max}=(1+\alpha)\bar{P}_{D_i},$$
$$i=1,\ \ldots,\ \mathrm{BN} \tag{31}$$

$$IC_{ij}^{\min} = (1-\beta)I\bar{C}_{ij}, \quad IC_{ij}^{\max} = (1+\beta)I\bar{C}_{ij},$$
$$i = 1, \ldots, \mathrm{BN}-1, j = i+1, \ldots, \mathrm{BN} \qquad (32)$$

Γ_0 can adopt different values from 0 to 15 (i.e. the total number of candidate corridors holding uncertain investment costs) and Γ_i can adopt different values from 0 to 1 to control the robustness of the optimal solution concerning the uncertain electricity demand. Without loss of generality, the same Γ_i is assumed for all nodal power balance constraints. The results of deterministic expansion plan and robust expansion plan, when $\Gamma_0 = 15/2 = 7.5$, $\Gamma_i = 0.9$ and $\alpha = \beta = 0.05$, are tabulated in Tables 2 and 3, respectively. As depicted in these tables, three candidate corridors (i.e. 2-3, 4-6 and 5-6), providing 540 MW cumulative transmission capacity, are selected in the deterministic case whereas four candidate corridors (i.e. 1-3, 2-3, 4-6 and 5-6), holding 590 MW cumulative transmission capacity, are selected in the robust case. Distinctly, the uncertain electricity demands and investment costs impose 50 MW extra transmission capacity costing 12.3 M$ on the robust case compared to the deterministic case.

Table 2 Deterministic expansion plan for Garver test system

Corridor	Capacity, MW	Reactance, pu
2-3	3 × 80	3 × j0.05
4-6	3 × 50	3 × j0.04
5-6	3 × 50	3 × j0.04
expansion cost: 78 M$		

The impact of aforesaid uncertainties on the optimal expansion plan is demonstrated in Figs. 3 and 4 for different levels of uncertainty and conservation. Given $\Gamma_0 =$

Table 3 Robust expansion plan for Garver test system ($\Gamma_0 = 7.5$, $\Gamma_i = 0.9$, $\alpha = \beta = 0.05$)

Corridor	Capacity, MW	Reactance, pu
1-3	50	j0.04
2-3	3 × 80	3 × j0.05
4-6	3 × 50	3 × j0.04
5-6	3 × 50	3 × j0.04
expansion cost: 90.3 M$		

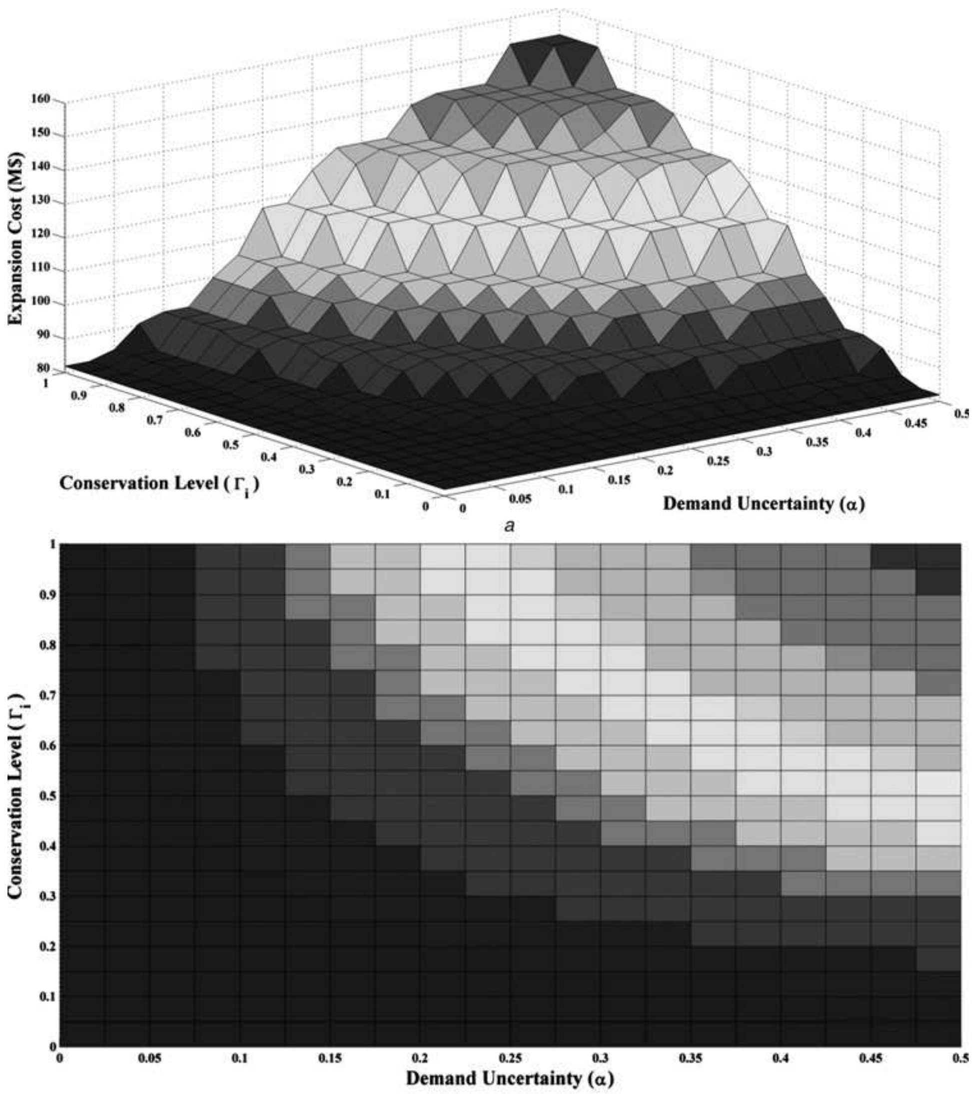

Fig. 3 *Expansion cost against Γ_i and α for $\Gamma_0 = 7.5$, $\beta = 0.05$*

a Side view
b Above view

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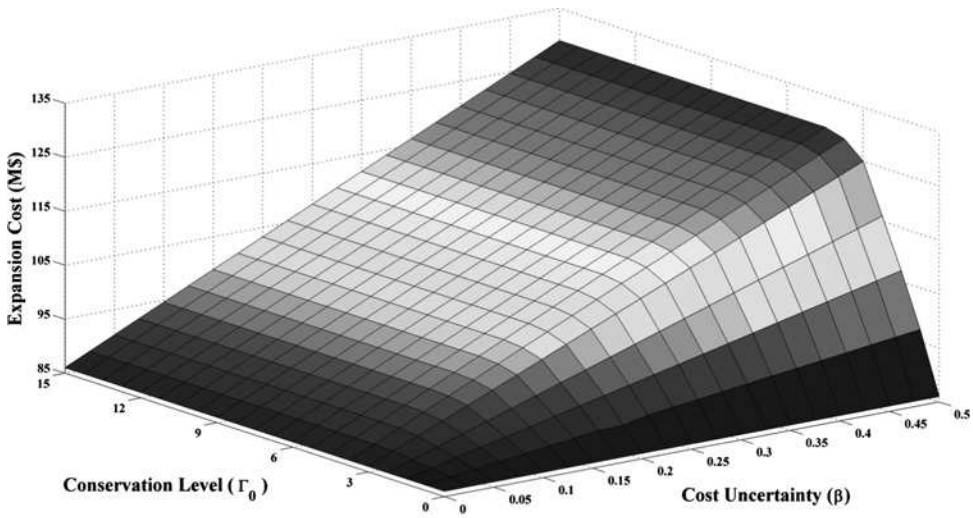
7.5 and $\beta = 0.05$, the expansion cost against α and Γ_i chosen by the planner is shown in Figs. 3*a* and *b* where different colours stand for different expansion costs. According to Fig. 3*b*, α has no effect on the planning results when Γ_i adopts different values equal or less than 0.1. It is also clear that for higher levels of conservation with low degrees of demand uncertainty (i.e. with the load forecast errors less than 2.5%) the results of the method do not take effect from Γ_i. However, the importance of RTEP methodology becomes vivid when α and Γ_i exceed 2.5% and 0.1, respectively. The same approach is surveyed to reflect the effect of the uncertain investment costs as illustrated in Figs. 4*a* and *b*. Distinctly, Fig. 4*a* shows that the expansion cost of RTEP approach (i.e. robust expansion cost immunised against the cost uncertainties) monotonically increases by increasing β (i.e. uncertainty of investment costs). As depicted in Fig. 4*b*, Γ_0 is only capable to control the conservation of total expansion cost regarding uncertain investment costs when it varies in the range of [0, 4] as only four candidate corridors are selected in the RTEP methodology.

4 Simulation results

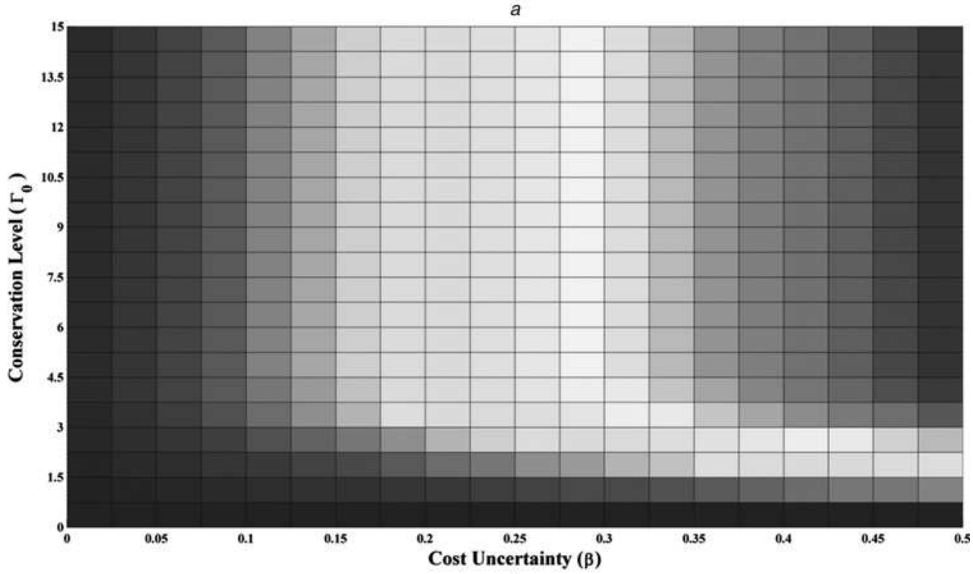
In this section, the simulation results are presented for IEEE 30-bus and IEEE 118-bus test systems [26] where the total generating capacity and total electricity demand for the both test systems are tripled. Also, $\alpha = \beta = 0.05$ and $\Gamma_i = 0.9$, the same as the illustrative example, are assumed for the both test systems. The characteristics of available transmission line technologies to expand the test systems are tabulated in Table 4.

4.1 IEEE 30-bus test system

This case study includes 53 candidate corridors to expand the transmission system of IEEE 30-bus test system using available transmission line technologies tabulated in Table 4. Consequently, the robustness of the optimal expansion plan regarding the uncertain investment costs can be controlled by varying the value of Γ_0 from 0 to 53. The optimal investment plans for deterministic and robust cases are demonstrated in Tables 5 and 6, respectively. In the RTEP results, it is assumed that $\Gamma_0 = 53/2 = 26.5$. The deterministic



a



b

Fig. 4 *Expansion cost against Γ_0 and β for $\Gamma_i = 0.9$, $\alpha = 0.5$*

a Side view
b Above view

Table 4 Available transmission line technologies

Type	Capacity, MW	Reactance, pu	Cost, M$
A	50	j0.04	8
B	80	j0.05	10
C	100	j0.07	14
D	120	j0.10	18
E	150	j0.13	20
F	200	j0.17	25
G	250	j0.20	30

Table 5 Deterministic results for IEEE 30-bus test system

Corridor	Capacity, MW	Reactance, pu
7-8	2 × 50	2 × j0.04
8-20	150	j0.13
11-18	200	j0.20
13-17	80	j0.05
18-19	100	j0.07
19-21	200	j0.20
20-22	80	j0.05
20-25	50	j0.04
25-26	50	j0.04
26-30	120	j0.10
expansion cost: 154 M$		

Table 6 Robust results for IEEE 30-bus test system ($\Gamma_0 = 26.5$, $\Gamma_i = 0.9$, $\alpha = \beta = 0.05$)

Corridor	Capacity, MW	Reactance, pu
7-8	3 × 50	3 × j0.04
8-20	150	j0.13
11-18	200	j0.20
13-17	80	j0.05
18-19	100	j0.07
19-21	200	j0.20
20-22	80	j0.05
20-25	50	j0.04
25-26	50	j0.04
27-30	150	j0.13
expansion cost: 172.2 M$		

case represents construction of 1130 MW newly added transmission capacity through 11 circuits in 10 corridors with a total cost of 154 M$. However, the RTEP methodology declares construction of 1210 MW newly added transmission capacity using 12 circuits in 10 corridors costing 172.2 M$. Therefore the robust case in comparison with the deterministic case enforces additional transmission capacity and expansion cost for a predefined level of conservation encountering aforementioned uncertainties.

4.2 IEEE 118-bus test system

As a total number of 194 candidate corridors are considered in this case study, Γ_0 can adopt different values from 0 to 194. In deterministic expansion plan, 36 circuits are added in 24 corridors with a total cost of 383 M$ to expand the transmission system of IEEE 118-bus test system as tabulated in Table 7. Also, the total transmission capacity added to the system is equal to 2760 MW. For this case study, the RTEP methodology with $\Gamma_0 = 194/2 = 97$ suggests 42 circuits in 25 corridors with a total cost of 482 M$ as demonstrated in Table 8. As a result, the total

Table 7 Deterministic results for IEEE 118-bus test system

Corridor	Capacity, MW	Reactance, pu
1-3	50	j0.04
10-11	3 × 50	3 × j0.04
22-24	80	j0.05
26-28	100	j0.10
31-39	120	j0.07
39-48	50	j0.04
40-52	50	j0.04
43-56	50	j0.04
45-46	2 × 80	2 × j0.05
53-58	100	j0.10
58-100	2 × 80	2 × j0.05
78-104	2 × 100	2 × j0.10
79-80	50	j0.04
82-87	2 × 80	2 × j0.05
89-93	80	j0.05
90-91	3 × 50	3 × j0.04
92-95	2 × 80	2 × j0.05
96-99	120	j0.07
102-105	80	j0.05
106-110	2 × 50	2 × j0.04
110-115	100	j0.10
112-114	200	j0.20
114-116	3 × 80	3 × j0.05
115-117	50	j0.04
expansion cost: 383 M$		

transmission capacity added to the system using the robust expansion plan is equal to 3320 MW. Distinctly, there is a large difference between the expansion costs of deterministic and robust expansion plans for IEEE 118-bus test system (i.e. 99 M$) as compared with IEEE 30-bus test system (i.e. 18.2 M$). The fact highlights the significant effect of the uncertain electricity demands and the uncertain investment costs on the expansion costs as the power system grows larger. It is to be noted that there is not much difference between the selected corridors in Tables 7 and 8. However, in the presence of uncertainty, the system is

Table 8 Robust results for IEEE 118-bus test system ($\Gamma_0 = 97$, $\Gamma_i = 0.9$, $\alpha = \beta = 0.05$)

Corridor	Capacity, MW	Reactance, pu
1-3	50	j0.04
10-11	4 × 50	4 × j0.04
22-24	80	j0.05
26-28	100	j0.10
31-39	120	j0.07
36-44	80	j0.05
39-48	50	j0.04
45-46	2 × 80	2 × j0.05
53-58	100	j0.10
58-100	3 × 80	3 × j0.05
63-69	120	j0.07
66-116	120	j0.07
78-104	2 × 100	2 × j0.10
79-80	50	j0.04
82-87	3 × 80	3 × j0.05
89-93	2 × 80	2 × j0.05
90-91	4 × 50	4 × j0.04
92-95	2 × 80	2 × j0.05
96-99	120	j0.07
102-105	80	j0.05
106-110	2 × 50	2 × j0.04
110-115	100	j0.10
112-114	200	j0.20
114-116	3 × 80	3 × j0.05
115-117	50	j0.04
expansion cost: 482 M$		

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Table 9 Violation chance of nodal power balance constraints using a priori and MCS methods

Γ_i	Maximum number of violations obtained by MCS methodology	Number of violated buses obtained by MCS methodology	Probability of violations	
			A priori estimate	MCS estimate
0.3	3388	4	0.1841	0.1694
0.6	99	1	0.0359	0.0050
0.9	0	0	0.0035	0.0000

reinforced with more circuits to deal with the probable electricity demand on the system buses.

It is interesting to justify the chance of constraint violation of the current study for IEEE 118-bus test system using the MCS methodology and compare it with the results obtained from (30) where electricity demands P_{D_i} $(i = 1, \ldots, 118)$ follow a normal probability distribution. For this purpose, the uncertain electricity demands are randomly generated and the following minimum virtual generation increment (MVGI) problem is solved for each sample of MCS (see (33))

Since L^*_{ij}, has already been evaluated from (29), the MVGI formulation (33) represents a simple LP problem. Then, the number of nodal power balance violations $(\varepsilon r_i \neq 0)$ is calculated. In this study, the mean value μ_{D_i} and the standard deviation σ_{D_i} of electricity demand for each bus are assumed to be $\bar{P}_{D_i}$ and $(0.05/3) \cdot \bar{P}_{D_i}$, respectively.

To guarantee the accuracy of MCS analysis based on definition of variation coefficient [24], 2×10^4 simulations are performed in this study maintaining the variation coefficient less than 1%. Table 9 represents chance of violation of nodal power balance constraints for different levels of conservation using analytical and MCS estimates. For instance, a priori chance of violation for power balance constraints with $\Gamma_i = 0.9$ can be calculated using (30) as follows (see equation at the bottom of the page)

Despite the chance of violation is estimated to be equal or less than 0.0035, the MSC estimate for $\Gamma_i = 0.9$ is zero. Hence, the MSC results confirm that a priori chance of violation for nodal power balance constrains represented by (30) presents an upper bound for violation chance. This provides a practical tool to select the value of Γ_i before applying the proposed method.

5 Conclusion

It is always desirable to allocate financial resources in the most economical way throughout the planning procedures. However, there is usually a typical challenge in this process for the long period of planning horizon, and that is the uncertainty. In this survey, investment costs and electricity demands are considered as two main sources of uncertainty in the planning studies. A linear TEP model is specifically designed to include different constraints and a robust optimisation framework is developed to deal with the mentioned uncertainties. It is shown that the method can be used in practical power systems. The results pinpoint the necessity of an accurate load forecast for an economically proper expansion plan. Further research works can be concentrated on considering the uncertainty source associated with unavailability of transmission lines in the proposed TEP approach to further ensure reliability of the expanded system.

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$$
\begin{aligned}
&\text{Min} \quad \sum_{i=1}^{\text{BN}} \varepsilon r_i \\
&\text{s.t.} \quad \sum_{\substack{j=1 \\ j \neq i}}^{\text{BN}} \left[\left(B_{ij}^{\text{prex}} - L_{ij}^* \, \Delta B_{ij} \right) \left(\theta_j - \theta_i \right) \right] = P_{G_i} - P_{D_i} + \varepsilon r_i, \quad i = 1, \ldots, \text{BN} \\
&\qquad \left| - \left(B_{ij}^{\text{prex}} - L_{ij}^* \, \Delta B_{ij} \right) \left(\theta_i - \theta_j \right) \right| \leq F_{ij,\max}^{\text{poex}}, \quad i = 1, \ldots, \text{BN} - 1, \quad j = i + 1, \ldots, \text{BN}
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
P\left[\sum_{\substack{j=1 \\ j \neq i}}^{\text{BN}} \left[B_{ij}^{\text{prex}} \left(\theta_j - \theta_i \right) + \left(\Psi_{ji} - \Psi_{ij} \right) \right] \neq P_{G_i} - \tilde{P}_{D_i} \right] &\leq 1 - \Phi \left[\frac{\left(\bar{P}_{D_i} + \Gamma_i \, \hat{P}_{D_i} \right) - \mu_{D_i}}{\sigma_{D_i}} \right] \\
&\leq 1 - \Phi \left[\frac{\left(\bar{P}_{D_i} + 0.9 \times 0.05 \times \bar{P}_{D_i} \right) - \bar{P}_{D_i}}{\dfrac{0.05 \times \bar{P}_{D_i}}{3}} \right] \\
&\leq 1 - \Phi(2.7) = 0.0035
\end{aligned}
$$

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7 Appendix

To elucidate the role of big M linearisation technique, suppose that the maximum number of circuits which can be added from bus i to bus j is equal to 2 (i.e. $L_{ij,\max} = 2$). Therefore the integer variable L_{ij} can adopt the values 0, 1 or 2. Owing to multiplication of L_{ij} and θ_j, $L_{ij}\ \Delta B_{ij}\ \theta_j$ is a non-linear term. The non-linearity of $L_{ij}\ \Delta B_{ij}\ \theta_j$ can be resolved using a continuous auxiliary variable (i.e. Ψ_{ij}) as well as two binary variables (i.e. ξ_{ij}^1 and ξ_{ij}^2) modelling different states of L_{ij} (i.e. 0, 1 and 2) as given in (5)–(10). Based on (10), either both of ξ_{ij}^1 and ξ_{ij}^2 are zero or one of them is one and the other is zero. Thus, the following states can be considered for Ψ_{ij} regarding the values adopted by L_{ij}

State 1: $L_{ij} = 0$ meaning no new circuit is added from bus i to bus j. In this state, (9) results in

$$L_{ij} = \sum_{l=1}^{2} l\ \xi_{ij}^l \rightarrow L_{ij} = 1\ \xi_{ij}^1 + 2\xi_{ij}^2 \rightarrow 0 = 1\ \xi_{ij}^1 + 2\ \xi_{ij}^2$$
$$\rightarrow \xi_{ij}^1 = 0, \quad \xi_{ij}^2 = 0$$

Consequently, Ψ_{ij} will be enforced on zero based on (7) and (8)

$$\left.\begin{aligned} \Psi_{ij} &\le M\left(\xi_{ij}^1 + \xi_{ij}^2\right) = M(0+0) = 0 \\ \Psi_{ij} &\ge -M\left(\xi_{ij}^1 + \xi_{ij}^2\right) = -M(0+0) = 0 \end{aligned}\right\} \rightarrow \Psi_{ij} = 0$$

Also, the original non-linear term $L_{ij}\ \Delta B_{ij}\ \theta_j$ becomes zero in this state. In this case, constraints (5) and (6) for $l = 1, 2$ are neutralised (note that M is a sufficiently large positive number).

State 2: $L_{ij} = 1$ meaning one new circuit is added from bus i to bus j. In this state, (9) results in

$$L_{ij} = \sum_{l=1}^{2} l\ \xi_{ij}^l \rightarrow L_{ij} = 1\ \xi_{ij}^1 + 2\ \xi_{ij}^2 \rightarrow 1 = 1\ \xi_{ij}^1 + 2\ \xi_{ij}^2$$
$$\rightarrow \xi_{ij}^1 = 1, \quad \xi_{ij}^2 = 0$$

Consequently, Ψ_{ij} will be enforced on $1\ \Delta B_{ij}\ \theta_j$ based on (5) and (6) for $l = 1$

$$\left.\begin{aligned} &\Psi_{ij} + M\ \xi_{ij}^1 \le 1\ \Delta B_{ij}\ \theta_j + M \xrightarrow{\xi_{ij}^1 = 1} \Psi_{ij} + M\ 1 \le 1\ \Delta B_{ij}\ \theta_j + M \\ &\Psi_{ij} - M\ \xi_{ij}^1 \ge 1\ \Delta B_{ij}\ \theta_j - M \xrightarrow{\xi_{ij}^1 = 1} \Psi_{ij} - M\ 1 \ge 1\ \Delta B_{ij}\ \theta_j - M \end{aligned}\right\}$$
$$\rightarrow \Psi_{ij} = 1\ \Delta B_{ij}\ \theta_j$$

Also, the original non-linear term $L_{ij}\ \Delta B_{ij}\ \theta_j$ becomes $1\ \Delta B_{ij}\ \theta_j$ in this state. In this case, constraints (5) and (6) for $l = 2$ as well as constraints (7) and (8) are neutralised.

State 3: $L_{ij} = 2$ meaning two new circuits are added from bus i to bus j. In this state, (9) results in

$$L_{ij} = \sum_{l=1}^{2} l\ \xi_{ij}^l \rightarrow L_{ij} = 1\ \xi_{ij}^1 + 2\ \xi_{ij}^2 \rightarrow 2 = 1\ \xi_{ij}^1 + 2\ \xi_{ij}^2$$
$$\rightarrow \xi_{ij}^1 = 0, \quad \xi_{ij}^2 = 1$$

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Consequently, Ψ_{ij} will be enforced on $2\,\Delta B_{ij}\,\theta_j$ based on (5) and (6) for $l = 2$

$$\left.\begin{array}{l}\Psi_{ij} + M\,\xi_{ij}^2 \le 2\,\Delta B_{ij}\,\theta_j + M \xrightarrow{\xi_{ij}^2=1} \Psi_{ij} + M\,1 \le 2\,\Delta B_{ij}\,\theta_j + M \\ \Psi_{ij} - M\,\xi_{ij}^2 \ge 2\,\Delta B_{ij}\,\theta_j - M \xrightarrow{\xi_{ij}^2=1} \Psi_{ij} - M\,1 \ge 2\,\Delta B_{ij}\,\theta_j - M\end{array}\right\}$$
$$\rightarrow \Psi_{ij} = 2\,\Delta B_{ij}\,\theta_j$$

Also, the original non-linear term $L_{ij}\,\Delta B_{ij}\,\theta_j$ becomes $2\,\Delta B_{ij}\,\theta_j$ in this state. In this case, constraints (5) and (6) for $l = 1$ as well as constraints (7) and (8) are neutralised. Thus, it is shown that the implemented big M linearisation technique can correctly model all possible values of the non-linear term (i.e. $L_{ij}\,\Delta B_{ij}\,\theta_j$) for $L_{ij,\max} = 2$. For the other values of $L_{ij,\max}$, (4)–(10) can similarly model the non-linearity of Ω_{nlt} by the linear formulation based on continuous and binary auxiliary variables.